LETTERS TO THE EDITOR

TO THE EDITOR

A recent R & D Note by Luus and Jaakola (1973) references a plant that has been a choice example for study by different optimal research schemes. The primary reference for defining the degrees of freedom for the material balance equations and their ordering and substitution of variables suggested by Luus (1937b) is Christiansen (1970) who disposed of this question for a general class of equations of interest in optimal plant design. The fact that the optimal location was unaffected by changes in FP for the steady state plant was indicated by Williams and Otto (1960) and reiterated by Mason (1971) and Mason and Crosser (1972).

We list all references to papers using this plant which we know of so that other workers may use results previously obtained and published. Figure 1 shows the *ROIS* for constant *FRC* and *T* for the investment used by Adleman (1972), Alghren (1966), Dibella (1965), Luus (1973), Ray (1971), and Vinturella (1968) for *FP* = 4763. Important singularities are the steep cliff near $FG = 0$ ($FG \geqq 0$ up to the right), the singularity located by the broken line, and the steep negative lobe located between them. Search procedures can be confused by the steep ridge or obtain the undesirable negative lobe which is positive for other choices of equations for the investment (Mason, 1971). At lower values of *T* and *FRC* the ridge maximum shifts along the cliff to the left, but the surface shape remains as shown. We especially feel that the plots (Figure 1) of the response surface will be of value to those who wish to understand the relative success of optimal search schemes for this plant and to realize that the location of the optimum is sensitive to the correlation used for the investment.

Because the equations can be ordered into a straight-forward (direct) calculational scheme, the equation for the *ROI* can be obtained in terms of the set of 4 (or 5 if *FP* is included) independent variables. While the expression is lengthy, and a reduced form depends upon the correlation chosen for the investment, we have studied two cases in some analytical detail.

In case I studies, the investment is assumed to be constant, which corresponds to the case of an operating plant already constructed. This turns out to be interesting in addition because *CHESS-UMR* simulations show the investment changes less than 10% over the range of positive return. The resulting equation, which shows the net profit, for constant *FRC* and *T* is

$$10^{-8}R = \frac{2.71\phi^3y^3 + (-.737\phi^3 + 9.35\phi^2)y^2 + (.850\phi^2 - 7.35\phi - .034\phi^3)y - .253\phi + .04\phi^2}{y(y\phi - 1.19)} \qquad (1)$$

where $y = 10^{-4}$ *FRE*, $\phi = FD/FS$ (unity minus the flow recycle ratio). This is shown for $R > 0$ in Figure 2, where the presentation is simplified by replacing the original coordinates of Figure 1 with $m = \phi y$ as a function of ϕ. The curved ridge of Figure 1 then becomes essentially horizontal as shown in Figure 2.

The singularities at $y = 0$, $y\phi = 1.19$ remind us that the chemical reactions of this plant are irreversible and feasible solutions must be at larger values of y and ϕy. This surface shows a max at $\phi = 0$ corresponding to total recycle of all byproduct (practical considerations suggest that operating expense might permit this).

Case II studies include some kind of investment (*I*) correlation, such as that of Dibella and Stevens (1965) or Mason (1971). Although the location of the optimum is altered by the *I* correlation, the analysis is unaffected, and only the details for investment proportional to *FR* (Mason (1972)) will be presented. For this case at constant *T* and *FRC* and for $\phi = .4$ the equation is

$$\frac{-y^3 + 3.340y^2 - 2.580y - 0.0355}{y^3 - 1.140y^2 - 0.041y + 0.0965} = 2.24\,Z \qquad (2)$$

this ratio of cubics can be resolved to

$$\frac{0.559}{y + 0.270} + \frac{0.480}{y - 0.320} - \frac{0.0425}{y - 1.097} = Z \qquad (3)$$

where $Z = 10^{-4}$ *ROI*, $y = 10^{-4}$ *FRE*, *PHI* = 0.4, *FRC* = 1500 lb/hr, and $T = 580$ *R* which illustrates the singularities and adjacent max pictured in Figure 1.

The behavior of the surface in *FRC* and *T* is contained in the coefficients of Equation (2). Straight-forward analysis, for Equation (1), of the most important terms of the polynomial shows that the extreme must lie at the lowest *T* allowed and for the lowest value of *FRC*. Although the maximum is unattractive physically, it is an excellent point from which to begin the optimal search of a more complete representation such as that of *CHESS*.

Approximations, simulations, and analysis of this kind are valuable when they reduce the total time and expense of calculation. Indeed, when Whitehead (1973) maintains $FB \geqq 0$ in his calculations for the material balance, he defines an even narrower region on this surface. The dominant contribution to the ridge is the numerator of Equation (2) indicating that it is the net profit, and not the investment, which is the controlling feature of this plant.

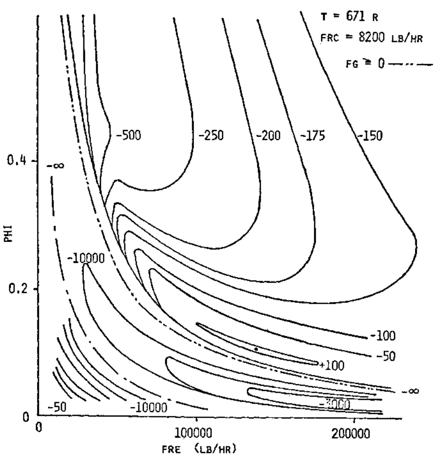

Fig. 1. Contours of return on investment—Adleman (1972).

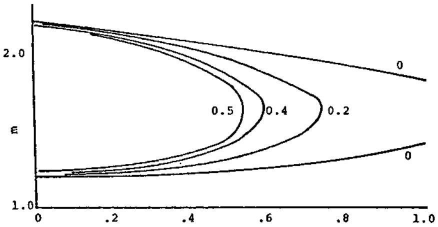

Fig. 2. Contours of revenue as function of *m* and ϕ.

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TO THE EDITOR

In a recent Letter to the Editor, Dyer (1973) has as a "main objective . . . to set straight the common error made in using Fick's law for sublimation dehydration (freeze-drying) problems." He maintains that this error is failure to account for the bulk flow due to a gradient in total pressure and indicates that this omission has been made by Sandall et al. (1967).

Publication delays may have obscured the reasoning behind the mass-transfer equation used by Sandall et al. Their effective internal mass-transfer coefficient D′ is based upon concurrent work by Gunn and King (1969, 1971). The general expression presented as Equation (8) by Gunn and King (1969) allows fully for bulk flow due to a gradient in total pressure, as well as other viscous, diffusive, and Knudsen flow effects. An entirely equivalent expression is given by Mason et al. (1967) in a very different form. Both Mason et al. and Gunn and King demonstrate experimental verification of the analysis over a wide range of conditions covering gradients in both total pressure and composition.

Gunn and King (1971) showed that this general equation could be greatly simplified for freeze-drying of meat, yielding Equation (16) of Sandall et al. (1967) for conditions where the rate-limiting effects of mass transfer become important. Data presented in Figure 10 of Sandall et al. bear out the form of this simplified equation. A survey of the available published transport data for freeze-dried foods convinced the authors that this approximation is valid for a much wider range of freeze-dried products than just poultry meat. However, conceivably drying conditions and transport coefficients may occur for which the simplified relationship is invalid. In this case it is only necessary to utilize the general relationship referred to above at the cost of considerable increased complexity.

It has been shown (Gunn and King, 1971) that experimental freeze-drying rate curves for meat can be predicted with good accuracy using the simplified equation. In these calculations only one constant, the external mass transfer coefficient, was fitted to the experimental drying data. The relevant transport coefficients were measured directly on the same samples of meat previously freeze-dried by Sandall et al. (1967). Effectiveness factors for diffusion were determined from the high pressure isobaric counter-diffusion of nitrogen-helium mixtures. Effective Knudsen diffusivities were determined from low pressure permeation measurements of pure nitrogen. This procedure is very different from that of force-fitting an equation with an adjustable, pressure-dependent transport coefficient to drying data. In the latter case an incorrect mathematical model can be fitted to the data, but the calculated transport coefficients will bear no relationship to those determined from direct experimental measurements.

The division between diffusional flow and bulk flow is somewhat arbitrary when Knudsen transport is a contributing factor. The definition of D_{te} adopted by Dyer (1973) is proper when bulk diffusion and viscous flow alone are involved, but results in D_{te} being dependent upon both the level and gradient of absolute pressure in a complex way when Knudsen flow contributes. Further, D_{te} cannot readily be related to fundamentally different independent measurements of transport coefficients, as can the parameters used in the analysis of Gunn and King. This essential difference is not related to inherent differences between the dusty-gas and capillary models of porous media. Wakao et al. (1965) obtain a prediction of simultaneous diffusion and flow from the capillary model which is close to the result of Gunn and King and Mason et al. for the dusty-gas model.

The conditions under which mass-transfer limitations become important in freeze-drying should also be considered. Mass transfer external to the food is neglected in the analyses given by Dyer and Sunderland (1968) and Cox and Dyer (1972) but can be a significant rate limit in the presence of substantial partial pressures of inerts and/or when there are constrictions in vapor flow paths within the chamber. Analysis of the simultaneous effects of mass and heat transport shows that mass transfer within the dried shell of the food becomes a significant rate limit for two conditions—for substantial partial pressures of inerts, in which case the simplified equation of Sandall et al. is valid, and for very low frozen-zone temperatures during drying, in which case the Knudsen diffusion term dominates with a significant contribution from viscous flow also being possible in some cases. If inerts are absent in the latter case, Equation (7) from Gunn and King (1971) is applicable.

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